Advantages of rear steer in LTI and LPV vehicle stability control

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Donald Selmanaj [1] Matteo Corno [1] Olivier Sename [2] and Sergio Savaresi [1]

Abstract— **In this paper, the advantages of the rear wheel steer in robust yaw stability control of four wheeled vehicles are shown. A MIMO vehicle dynamic stability controller (VDSC) involving front steer, rear steer and rear braking torques is synthesized. The comparison between a vehicle with and without rear steer is done on avoidance maneuver using both LTI and gain-scheduling LPV controller. Both robust H_∞ controllers are built by the solution of an LMI problem. To better evaluate the influence of the rear steer on the performance time domain indexes are introduced. The simulation results show that active rear steer enhances vehicle handling on a low friction surface.**

Index Terms— **LPV controller, four-wheel steering, rear braking torques, yaw stability control**

I. Introduction

In the last years active safety systems are widely spread in commercial light vehicles and several solutions to global chassis control can be found in literature. They can be classified by control structure and actuators used to ensure stability. The number of available actuators (control variables) is imposed by the mechanical layout.

In brake-based studies (see, e.g., [1], [2], [3]) the vehicle behavior is controlled through torque distribution to the four wheels. Brake-based solutions imply a relatively simple mechanical layout, however the induced vehicle behavior presents a strong dependence on the longitudinal velocity.

Furthermore solutions involving both braking and active front steering (AFS) have been proposed (see [4],[5]). The combined management of these actuators leads to improved vehicle handling and stability, however the interaction between the front active steer controller and the driver might influence the driveability of the vehicle.

To take full advantage of the tire grip, four wheel steering (4WS) architectures combined with brake-based architectures have been proposed. Mainly due to increased mechanical complexity, these solutions are not spread in commercial light vehicles, however many studies have evaluated the advantages introduced by an active rear steering. In [6] and [7] decoupling control architectures have been proposed in order to reduce the interaction between the yaw rate and lateral dynamics, while in [8], [9], [10] and [11] robust control architectures are introduced in order to overcome external disturbances, such as wind forces and parameter variations due to the running vehicle condition. Among those tire cornering stiffness is a key influencing factor on maneuverability.

The present work is an extension of the previous one in [12] and [13] where a collaborative control of active front steer and rear brake torques is proposed. Two kinds of controllers are implemented: an LTI (Linear Time Invariant) controller and an LPV (Linear Parameter-Varying) controller. The LTI controller uses all available control variables in every condition while the LPV controller allows to choose whenever activate or deactivate a control input. The Activation criteria can depend on the vehicle running condition or it can depend on a fault detection system. For instance, if a failure occurs and an input is not available anymore, the LPV controller can switch to a different configuration still ensuring the stability of the system. It is worth noting that an LTI controller does not guarantee the system stability and performance if an actuator fails.

Here the vehicle architecture is extended introducing the active rear steering (ARS). Afterwards a LTI and a LPV controller are designed in the H_∞ framework, and the performance of both controllers and both types of vehicles (with and without rear steering action) are compared in a critical driving condition. The aim of the work is to identify the advantages introduced by the rear steering action in the yaw rate stability control and to emphasize the differences between the 2 types of controllers.

II. Control architecture

The control structure is represented in Fig. 1 and derived from [13]. It is a two-layer architecture. In the first layer the VDSC generates the desired steering angles and wheel torques; in the second level the ABS actuators at the rear axle and the steering actuators generate the actual control values. The control strategy implements a yaw reference tracking philosophy where the desired yaw is generated by a nominal model (here a bicycle model but some steady state evaluations could be done). The front active steering input is superimposed to the driver steering input. Two VDSC controllers are designed: an LTI controller employing all the available actuation and a LPV gain scheduled controller designed to better coordinate the available actuation (as explained later). Both controllers are designed following the $\mathscr{H}_\infty$ paradigm applied to the following extended bicycle

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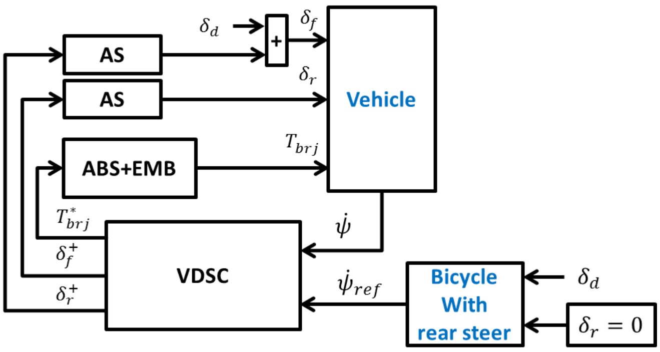

Fig. 1. Control architecture. AS are the active steering actuators and ABS+EMB are the wheel torque actuators (at the left/right rear wheels).

model:

$$\begin{bmatrix}\dot\beta\\ \dot\psi\end{bmatrix} = \begin{bmatrix}\mu\frac{-C_f-C_r}{mv} & 1+\frac{l_rC_r-l_fC_f}{mv^2}\\ \frac{l_rC_r-l_fC_f}{Iz} & \frac{-l_f^2C_f-l_r^2C_r}{vIz}\end{bmatrix}\begin{bmatrix}\beta\\ \dot\psi\end{bmatrix}$$

$$+\begin{bmatrix}\frac{C_f}{mv} & \frac{C_r}{mv} & 0 & 0 & 0\\ \frac{l_fC_f}{I_z} & \frac{-l_rC_r}{I_z} & \frac{1}{I_z} & \frac{C_\lambda t_r}{2I_z} & -\frac{C_\lambda t_r}{2I_z}\end{bmatrix}\begin{bmatrix}\delta_f\\ \delta_r\\ M_{dz}\\ T_{rl}\\ T_{rr}\end{bmatrix} \quad (1)$$

where β is the vehicle slip angle, $\dot\psi$ is the yaw rate, δ_f and δ_r are the front and the rear steering angles respectively, T_{rl}, T_{rr} are the rear breaking torques and M_{dz} is the yaw rate disturbance (lateral wind effect for instance).

Fig. 2 shows the chosen $\mathscr{H}_\infty$ control configuration, including

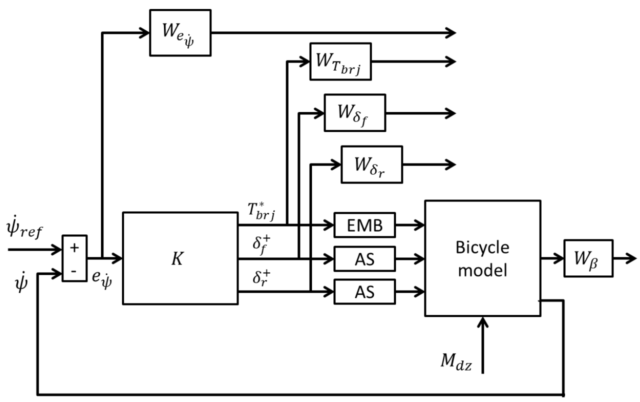

Fig. 2. Generalized plant used for synthesis.

some weighting functions that represent the desired closed-loop performances and handle the different limitations on the control inputs.

- $W_{T_{brj}}$ $(j = l, r)$ limits the use of the braking actuators at high frequencies ($10Hz$).
- $W_{\delta f}$ is a band-pass filter witch allows the control to act below the actuator bandwidth frequency ($10Hz$) and above the drivers acting range of frequencies (we consider $[0,1]Hz$).
- $W_{\delta r}$ penalizes the use of the rear steering actuator at high frequencies ($10Hz$).
- W_{e_ψ} ensures the tracking performances at low frequency
- W_β penalizes high side slip angles at low frequency.

It is worth noting that, in Fig. 2, the bicycle model and the weighting functions are LTI systems. Nevertheless K will be either an LTI controller as an LPV one. The first case naturally arises following the control configuration in Fig. 2. On the other hand an LPV controller will be designed imposing a specific parameter dependent structure to the controller state space representation (as presented in [12]).

A. LTI controller structure

The output feedback LTI controller serves as a benchmark. It is a fixed structure controller using all the control variables in every situation. Two different versions of the controller are designed, LTI1 uses front wheel steering and the brake actuators; LTI2 uses in addition the rear wheel steer. Fig.

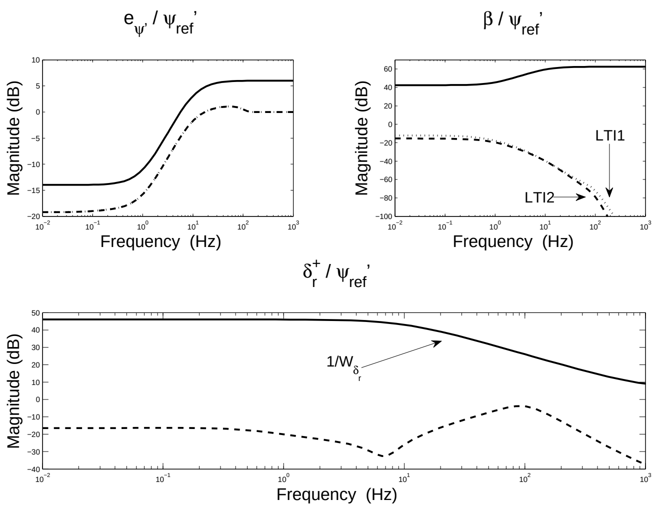

Fig. 3. LTI controller, frequency response: LTI1 (black dotted), LTI2 (black dashed). Weighting functions $1/W_{e_\psi}, 1/W_\beta, 1/W_{\delta_r^+}$ (black solid).

3 shows the sensitivity functions of the two closed loop systems (controller + bicycle model).The rear wheel steering (which concerns only the LTI2) does not seem to improve the closed loop nominal performances.

B. LPV controller structure and parameters choice

The LTI controller employs all the control variables indifferently. Therefore all the actuators are used simultaneously, which could be not optimal in an energetic sense, and could be too obscure for car manufactures since the use of the control actions are not related to the driving situations. As shown preliminary in [12] the LPV approach is a solution to a better coordination of the different control inputs (braking and steering) since it allows to activate the actuators only when requested by the driving situation (for instance the vehicle state behavior). Here the control strategy extends the previous works including the use of the rear steering actuators. An LPV controller is then designed here, whose parameters weight the use of the actuators. Indeed the resulting output feedback LPV controller is chosen of the form:

$$
K(\rho_1,\rho_2,\rho_3):
\begin{cases}
\dot{x}_c(t) = A_c(\rho_1,\rho_2,\rho_3)\,x_c(t) + B_c(\rho_1,\rho_2,\rho_3)\,e_{\dot{\psi}}(t) \\
\begin{bmatrix} \delta_f^+ \\ \delta_r^+ \\ T_{brl}^* \\ T_{brr}^* \end{bmatrix} = \underbrace{\begin{bmatrix} \rho_1 & 0 & 0 & 0 \\ 0 & \rho_2 & 0 & 0 \\ 0 & 0 & \rho_3 & 0 \\ 0 & 0 & 0 & (1-\rho_3) \end{bmatrix}}_{U(\rho)} C_c^0(\rho_1,\rho_2,\rho_3)\,x_c(t)
\end{cases}
\tag{2}
$$

Rem. Note that $U(\rho)$ is chosen and implies that K is LPV. If $U(\rho) = I_4$ the K will be LTI.

It depends on three parameters. ρ_1 and ρ_2 weight the front and rear steer respectively, and ρ_3 switches the torque action from the left rear to right rear wheel and vice versa. In the present work they take value 0 or 1 according to the following criteria:

- ρ_1: allows to activate the front steer ($0 \rightarrow$ no steering action, $1 \rightarrow$ full steering action);
- ρ_2: the rear steer is only activated in critical conditions, namely when the stability index ($S_{index} = \left|9.5\beta + 2.49\dot{\beta}\right|$) is above a threshold limit (if $S_{index} > 0.3$ $\rho_2 = 1$ otherwise $\rho_2 = 1$);
- ρ_3: handles the over/under steering situations. Depending on the sign of the yaw rate error only one of the two braking torque is allowed to act. Namely if $e_{\dot{\psi}} > 0 \rightarrow \rho_3 = 1$ otherwise $\rho_3 = 0$

The LPV controller structure (2) is very generic. It allows an adaptive use of the front and rear steering actions. Only the distribution of the left/right rear breaking torques is imposed by the value of ρ_3. In this paper a specific choise of the parameters values is considered to emphasize the additional use of the rear steering control action.

The LPV controller design problem can be cast into a set of LMI's [14] defined over the vertices of the polytope identified by the parameter space. The employed criteria effectively reduce the number of parameter to 2, yielding 4 vertices. Each vertex represent a specific combination of the parameters. The final LPV controller is a combination of four controllers, one for each of the vertices considered in the parameter choice. In this following analysis (in simulations) we have considered that the front steering is always active ($\rho_1 = 1$). Then the control is obtained as follows:

$$
\begin{aligned}
\begin{bmatrix} \delta_f^+ & \delta_r^+ & T_{brl}^* & T_{brr}^* \end{bmatrix}^T = & \rho_2\rho_3 K(,\overline{\rho_2},\overline{\rho_3}) \\
& + \rho_2(1-\rho_3)K(\overline{\rho_2},\underline{\rho_3}) \\
& + (1-\rho_2)\rho_3 K(\underline{\rho_2},\overline{\rho_3}) \\
& + (1-\rho_2)(1-\rho_3)K(\underline{\rho_2},\underline{\rho_3})
\end{aligned}
\tag{3}
$$

The set of LMI's are solved using SeDuMi and Yalmip [15]. For further information and details about the LMI optimization for H_∞ synthesis refer to [13] [16] [14] [17] [18]

On the other hand to more easily assess the advantages of rear wheel steering two controllers have been designed: LPV1 without rear wheel steering and LPV2 with rear wheel steering. Fig. 4 plots the sensitivity functions in the nominal design case for all vertices of the parameter space polytope. Two observation are in place:

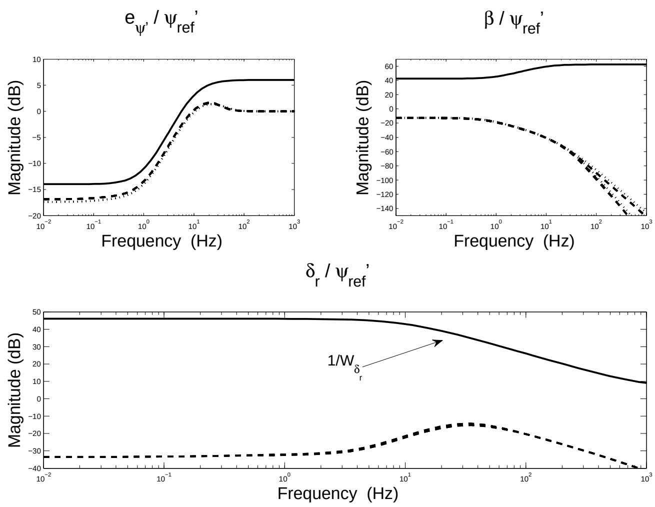

Fig. 4. LPV controller, frequency response: LPV1 (black dotted), LPV2 (black dashed). Weighting functions $1/W_{e_{\dot{\psi}}}, 1/W_\beta, 1/W_{\delta_r^+}$ (black solid)

- The fact that no difference can be observed among the sensitivity function at the vertices means that the LPV design is indeed successful in maintaining the specified performances throughout the parameter space.
- In the nominal case the addition of the rear wheel steering does not bring any advantage.

As long as frequency responses are concerned (of the closed loop system with LPV controllers and DRY road) we don't see any relevant differences between vehicle with and without rear steer.

III. Validation

In this section the proposed control strategies are validated in simulation using a full vehicle simulation model, whose parameters are described in [13] and [16] and have been validated on a real Renault Mégane vehicle. Unlike the bicycle model the full model includes a nonlinear tire characteristic, a nonlinear lateral and longitudinal dynamics together with a nonlinear vertical dynamics. For the purpose of this article the full model has been extended with the rear steer input. An obstacle avoidance maneuver is illustrated. The driver input is shown in Fig. 5 and the vehicle initial speed is $90km/h$. The simulation has been performed in 3 different road conditions: DRY ($\mu = 1$), WET ($\mu = 0.5$) and ICY ($\mu = 0.3$). This will emphasize the intrinsic robustness property of the proposed approach.

A. Avoidance maneuver on icy road

To better understand the advantages and differences caused by the use of the rear steering it is useful to analyze in details

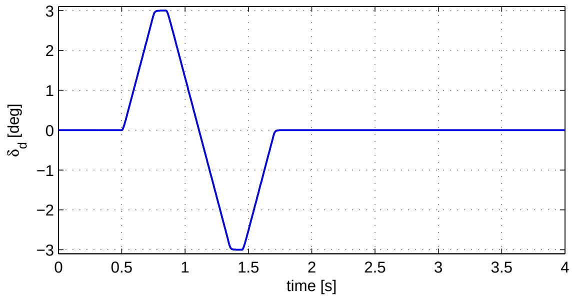

Fig. 5. Driver input: avoidance maneuver.

one maneuver in the time domain. The obstacle avoidance presented maneuver on icy road is this in what follows. Fig. 6 shows the absolute value of yaw rate error, vehicle velocity and the side-slip angle for the LTI case. Comparing the

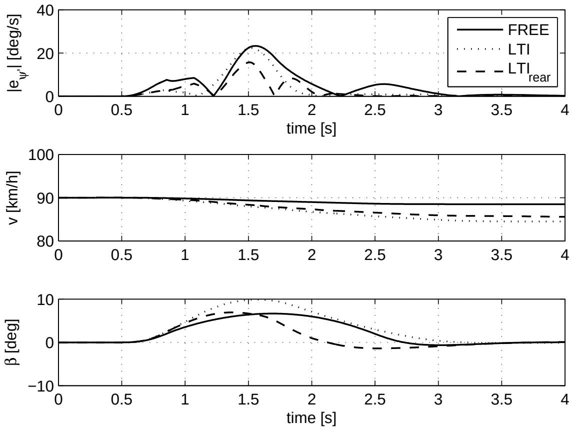

Fig. 6. Yaw rate error, speed and body slip angle: LTI (dotted), LTI with rear steer (dashed). Uncontrolled vehicle (black)

results of the LTI controllers with the ones of the LPV controller presented in Fig. 8 it is evident that the LTI controller achieves better performance. At the same time Fig. 7 and Fig. 9 show the huge difference in the actuator usage between the two types of controllers. While there is no relevant difference in the peak values reached by the wheels torques, the steering angles behavior between LTI and LPV controllers is highly different. It is worth noting that the LPV controllers through the coordination of the control actions, induce a reduced use of the actuators. Indeed the maximal rear steering angle reaches $3deg$ for the LPV with rear steering controller and $10deg$ for the LTI with rear steering. Moreover the front steering angle is less than $1deg$ for the LPV_{rear} controller and it reaches almost $3deg$ for the LTI_{rear}. Hence the interference with the driver action is also reduced thanks to the LPV front and rear steering strategy.

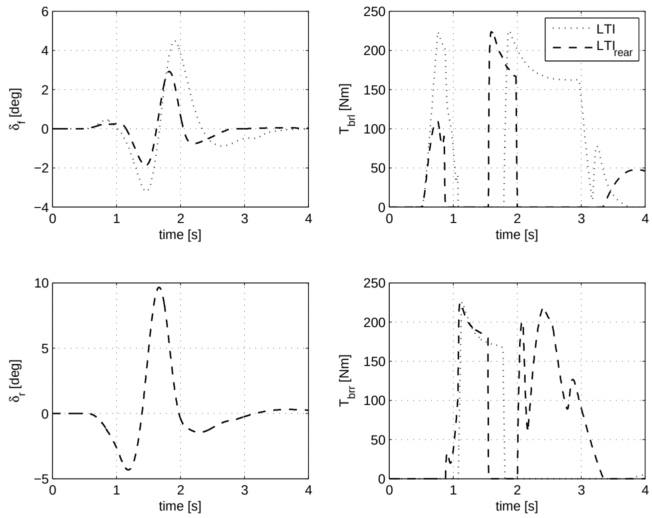

Fig. 7. Control inputs: LTI (dotted), LTI with rear steer (dashed).

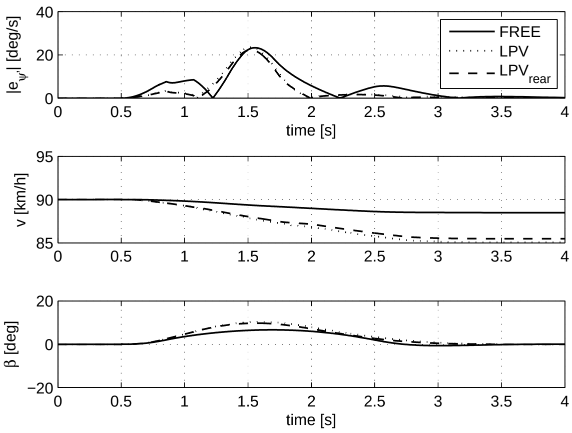

Fig. 8. Yaw rate error, speed and body slip angle: LPV (dotted), LPV with rear steer (dashed). Uncontrolled vehicle (black)

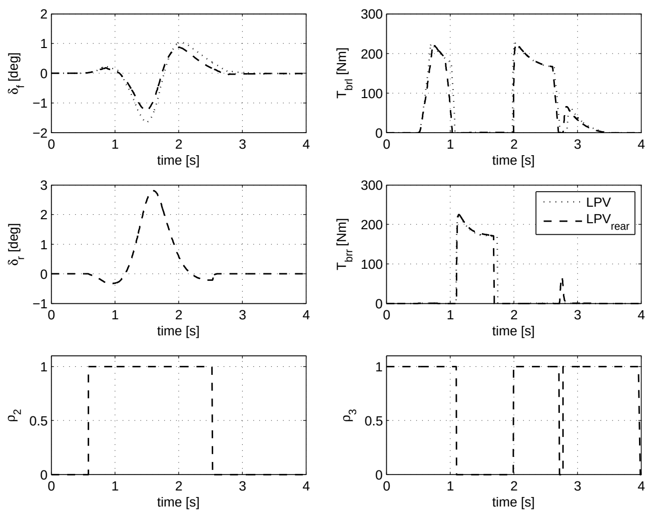

Fig. 9. Control inputs: LPV (dotted), LPV with rear steer (dashed).

B. Performance indexes

To better show the advantages of the rear steer we introduce three time domain indexes (4).

$$e_{\dot{\psi}_{index}} = \int_{t_0}^{t_{end}} |e_{\dot{\psi}}(t)| \, dt \tag{4a}$$

$$\beta_{index} = \max_t \beta(t) \tag{4b}$$

$$V_{index} = V_0 - \min_t (V(t)) \tag{4c}$$

The first index quantifies the yaw rate tracking error. The second index penalizes high side-slip angles. It is well known that non professional drivers cannot manage high side-slip angle. The third index penalized loss of velocity, ideally one would want to be able to stabilize the obstacle avoidance maneuver without reducing the vehicle velocity. For all performance indexes, a lower value is to be preferred. Fig. 10 and 11 show the simulation results of two vehicles (with and without rear steer) controlled by both LTI and LPV controllers in three different road conditions. The following comments can be drawn:

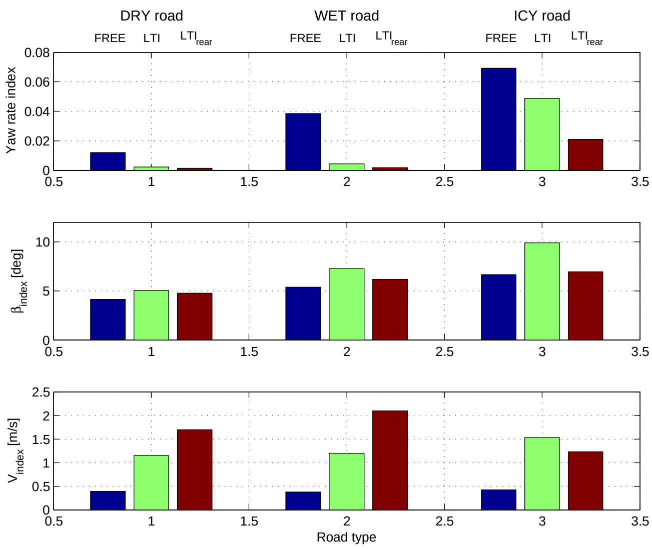

Fig. 10. LTI controller: indexes comparison

- All controllers considerably improve the yaw rate reference tracking in all conditions.
- All the controllers cause a velocity reduction at the end of the maneuver. This is mainly caused by the use of the brakes.
- In order to generate higher tire side slip (i.e. higher lateral tire force) all the controllers cause higher vehicle slip angle (in this case, the uncontrolled car is indeed not able to remain in the trajectory bound).
- As predicted by the sensitivity analysis, rear wheel steering does not bring any advantage on dry road.
- For both the LPV and LTI cases the use of rear wheel steering proves advantageous on low friction surfaces. The farther the vehicle is from the nominal design condition, the more an additional lateral dynamics control variable is useful.

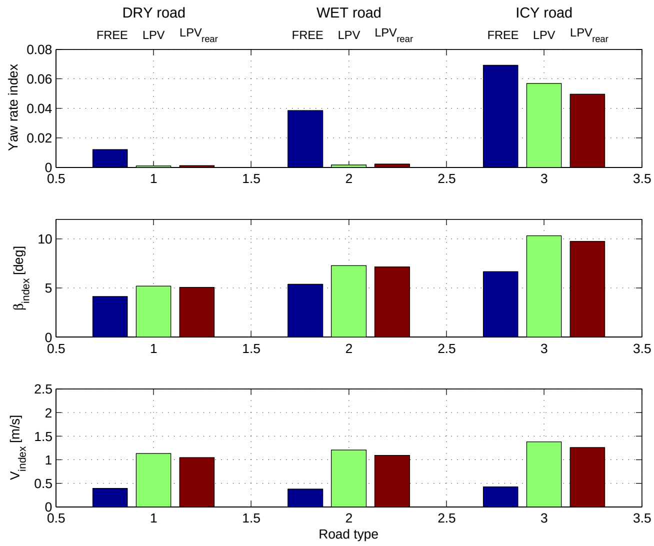

Fig. 11. LPV controller: indexes comparison

- It is also interesting to compare the LPV and LTI case. In particular, as expected, the LTI controllers offer better reference tracking performance. This is due to the fact that they can access to all control variables, however this freedom comes at the cost of reduced velocity at the end of the maneuver. The LPV controllers achieve a slightly worse reference tracking, but on the other side also a reduced loss of velocity. This is due to the retarded use of the braking actuator.

IV. CONCLUSIONS

In this paper the advantages for the rear wheel steering action on the global chassis control was studied. The work involved 2 kind of controllers: an LTI controller that uses all the actuator simultaneously and an LPV controller that switches between different configuration of the actuators in relation to the driving situation. The main results can be summarized as follows:

- The advantages of the rear steering action have been emphasized in the yaw control case with avoidance maneuver. Moving from a nominal condition (i.e. high grip road surface) to a more critical one (i.e. ICY road) the rear steering action becomes more relevant. It could be further emphasized in harder driving situations like braking and avoidance maneuver in a curve at high speed.
- Still with more complex architecture (like the one with rear steering) the LPV controller, compared to the LTI allows lower and customizable usage of the actuators leading to lower power consumption and lower interaction with the driver action.

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